PD-158

(521) M.Sc. PHYSICS (FIRST SEMESTER)

Examination DEC. 2020

Paper-I: Mathematical Methods-I

Time: Three Hours **Maximum marks: 80**

SECTION-A

1. Answer the following questions: **1×10**

(a) What do you mean by linearly dependent vectors?

(b) Define triangular matrix.

(c) State True or False: For any matrix A, AA^T and A^TA are always symmetric.

(d) State True or False: Rank of a matrix is the largest order of any non-vanishing minor of the matrix.

(e) Find the value of $\frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$.

(f) What is the value of $L_1(x)$.

(g) Bessel's function of second kind and order zero is known as____________.

(h) Find the Laplace transformation of $f(t)\delta(t-a)$.

(i) Write two advantages of Fourier series.

(j) Show that $x\delta'(x) = -\delta(x)$.

2. Answer the following questions: **2×5**

(a) Define dimension and basis of a vector space.

(b) Given $A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$. Check whether A is singular or not.

(c) Show that the necessary and sufficient condition for the existence of the inverse of a matrix ***B*** is that ***B*** is non-singular.

(d) Write the Dirichlet's conditions for a Fourier expansion.

(e) Find the value of $L\{t^3\delta(t-2)\}$.

SECTION-B

Answer the following questions: **15×4**

Unit-I

3. (a) Define vector space V_n over a field F.

(b) Show that the three-dimensional vectors $x^1 = \{2,0,-2\}$, $x^2 = \{0,2,0\}$, and $x^3 = \{2,0,2\}$ are linearly independent.

(c) Express the vector $\{0,0,3\}$ as a linear combination of the vectors x^i given above.

(d) If $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a set of n-independent vectors & β be a non-null vector such that $\beta = a_1\alpha_1 + a_2\alpha_2 + \cdots \dots a_n\alpha_n$, $(a_j \neq 0)$ then the vectors $\alpha_1, \alpha_2, \dots, \alpha_{j-1}, \beta, \alpha_{j+1}, \dots, \alpha_n$ are independent. **$6+2+2+5$**

OR

(a) What do you mean by symmetric and skew-symmetric matrix?

(b) Solve the following system of equations by matrix method:

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

5 + 10

Unit-II

4. Obtain the generating function of Legendre's polynomial $P_n(x)$

$$(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$$

and hence derive

(a) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

(b) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$ **9 + 6**

OR

Deduce the differential formula for Laguerre polynomial $L_n(x)$ and hence find the value of $L_2(x)$ & $L_3(x)$. **10 + 5**

Unit-III

5. (a) Write Bessel's differential equation of order n and find the complete solution when n has an integral value.

(b) Find the value of $J_{-1/2}(x)$. **10 + 5**

OR

Prove the orthogonality property for Hermite polynomial. **15**

Unit-IV

6. (a) Define Laplace transform and discuss its linearity and translation properties.

(b) Using Laplace transform, evaluate $\int_0^\infty e^{-t}\left(\frac{\cos at - \cos bt}{t}\right) dt$ **10 + 5**

OR

(a) Expand $f(x) = \sin x, 0 < x < \pi$ in Fourier cosine series.

(b) Evaluate $\int_{-\infty}^{+\infty} e^{-x}\delta(x^3 - x)dx$ **10 + 5**