AH 1543 CV-19
M.A./M.Sc. (Previous)
Term End Examination, 2019-20
MATHEMATICS
Paper- VI

Time:- Three Hours] [Maximum Marks:100

Note: Answer **all**questions. All Question carry equal marks.

1. (a) Define differentiable manifold and show that the real projective space PR^n is differentiable manifold.
   (b) Let M be a topological manifold with aC^∞ −atlas A. Then there exists a unique differentiable structure D containing A.

2. (a) Show that the Cartesian products of differentiable manifolds are again differentiable manifolds.
   (b) Show that open ball $B^n \subset R^n$is diffeomorphicto R^n.

3. Given the set $M = \{(P_1, P_2, P_3) \in R^3 : P_1^3 + P_2^3 + P_3^3 - 3P_1P_2P_3 = 1\}$
   (i)Show that M is smooth embedded manifold in R^3 of dimension 2.
   (ii) Compute TpM at P = (0,0,1).

4. (a) Define lie group and show that R^n is lie group with the group operation given by addition.
   (b) At ϕ:h→H be a smooth homomorphism of lie group. Then show that $\emptyset' = Te\ \emptyset : g = TeH \rightarrow h = TeH$ is lie algebra homomorphism.

5. (a) Show that for exterior darivetive
   $$d^2 = dod = 0$$
   (b) Prove that
   $$X_y = [T_yU]^{-1}.T_yU.X_y$$

6. State and prove generalized hauss and MainardiCodazzi equation.

7. (a) Prove that every vector Bundle E→B admits a connection.
   (b) State and prove Schur's theorem.

8. (a) Define almost complex manifolds and prove that C^n is an almost complex manifold.
   (b) Define principle fibre bundle and show that a principle fibre bundle admits a section if and only if it is trivial.

9. (a) Prove that Riemannian manifold (M,g) is geodesically complete if and only if D(p) = TpM
   (b) Prove that A Riemannian manifold (M,g) is connected if and only if any two point of M Can be joined by a broken geodesic.

10. (a) Show that tangent bundle is a vector Bundle.
    (b) Prove that $X^T = KMOTX$ for vector Bundle Homomorphism.