



**AI-1543**

**M. A./M. Sc. (Previous)**  
**Term End Examination, 2020-21**

**MATHEMATICS**

*Paper : Sixth*

**(Differential Geometry of Manifolds)**

*Time Allowed : Three hours*

*Maximum Marks : 100*

*Minimum Pass Marks : 36*

*Note : Answer any five questions. All questions carry equal marks.*

1. State and prove Gause's formulae.
2. State and prove sectional curvative schur's theorem.
3. State and prove Weingarten equations line of curvature.

4. (a) Prove that every manifold can be embedded into  $R^k$  for  $k$  sufficiently large.  
 (b) Show that tangent bundle is a vector bundle.
5. (a) Define topological group. Let  $f : h \rightarrow H$  be a homomorphism between topological group prove that  $f$  is continuous if and only if  $f$  is continuous at  $1 \in G$ .  
 (b) Prove that a closed subgroup  $H$  of a Lie group  $G$  is a Lie group with relative topology.
6. (a) Prove that a Lie group has no small subgroups.  
 (b) Define following with example :  
 (i) Riemannion manifolds  
 (ii) Lie transformation groups
7. (a) Prove that any vector bundle over the affine space  $R^k$  is trivial.  
 (b) Every vector bundle  $E \rightarrow B$  admits a connection.
8. (a) Show that a principal fibre bundle admits a section if and only if it is trivial.

(b) The nundle map  $f': \xi' \rightarrow f^* \xi$  is an isomorphism iff  $\bar{f}$  restrict to an isomorphism on each other.

9. (a) Show that for

$$d: \Omega_k(M) \rightarrow \Omega^{k+1}(M)$$

$$d^2 = d \circ d = 0$$

(b) Prove that :

$$D(1) = 2D(1)$$

10. State and prove Mainard-Codazzi equations.