PD-153-S.E.-CV-19

M.A./M.Sc. MATHEMATICS (1st Semester)

Examination, Dec.-2020

ADVANCED ABSTRACT ALGEBRA (I)

Time : Three Hours] **[Maximum Marks : 80**

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. **Answer the following questions:-** 1x10=10
   (a) What is the order of alternating group A_n .
   (b) Define odd and even permutations.
   (c) Define centre of a group what is the centre of abelian group.
   (d) Define n^{th} derived set of a group.
   (e) Define n^{th} centre of a group.
   (f) Give an example which show solvable group is not abelian.
   (g) When two subnormal series are called isomorphic or equivalent.
   (h) Give example of nil ideal.
   (i) Write a maximal ideal of Z.
   (j) Give example of prime ideal.
2. **Answer the following questions:-** 2x5=10
   (a) Show that Kernel of a group is normal subgroup.
   (b) Show that centre is a normal subgroup of a group.
   (c) Show that every nilpotent group is solvable but converse need not be true.
   (d) Show that Jordan-Holder theorem implies fundamental theorem of arithmetic.
   (e) Prove that every nilpotent ideal is necessarily nil.

Section-B 12x5=60

Answer all questions.

3. Show that every group is isomorphic to permutation group.

   OR

   Define inner automorphism and show that inner automorphism is a normal subgroup of automorphism of G.
4. Show that if a subgroup H and a quotient group G/H are solvable than G is solvable.

   OR

   Let G be the group and G' be its commutator subgroup then show that
   (a) G' is normal subgroup of G
   (b) for any normal subgroup H of G
   G/H is abelian if and only if $G' \subseteq H$
5. State and prove Jordan-Holder theorem.

   OR

   Prove that if G is a commutative group having a composition series then G is abelian.
6. Prove that an ideal M of a commutative ring R with unity is a maximal ideal if and only if $\frac{R}{M}$ is a field.

   OR

   If R is finite nonzero commutative ring then show that every maximal ideal is a prime ideal but converse need not be true. .
7. State Zorn's lemma and show that if T is a nonzero ring with unity 1, and I is an ideal of R such that $I \subseteq R$, then there exist a maximal ideal M of R such that $\subseteq$ M .

   OR

   Give an example to show that nil ideal does not imply nilpotent.